

Gluon Propagator in Fractional Analytic Perturbation Theory

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We consider the gluon propagator in the Landau gauge at low spacelike momenta and with the dressing function $Z(Q^2)$ at the two-loop order. We incorporate the nonperturbative effects by making the (noninteger) powers of the QCD coupling in the dressing function $Z(Q^2)$ analytic (holomorphic) via the Fractional Analytic Perturbation Theory (FAPT) model, and simultaneously introducing the gluon dynamical mass in the propagator as motivated by the previous analyses of the Dyson-Schwinger equations. The obtained propagator has behavior compatible with the unquenched lattice data ($N_f = 2 + 1$) at low spacelike momenta $0.4 \text{ GeV} < Q \lesssim 10 \text{ GeV}$. We conclude that the removal of the unphysical Landau singularities of the powers of the coupling via the (F)APT prescription, in conjunction with the introduction of the dynamical mass $M \approx 0.62 \text{ GeV}$ of the gluon, leads to an acceptable behavior of the propagator in the infrared regime.

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I. INTRODUCTION

The perturbative approach in QCD (pQCD) is known to work well at high momenta ($|q^2| \gtrsim 10 \text{ GeV}^2$). However, at low momenta ($|q^2| \sim 1 \text{ GeV}^2$) it is unreliable, principally because the pQCD coupling parameter $a_{\text{pt}}(Q^2) \equiv \alpha_s(Q^2)/\pi$ has (Landau) singularities at spacelike low momenta q : $0 < Q^2 (\equiv -q^2) \lesssim 10^{-1} \text{ GeV}^2$. This singularity structure is incompatible with the analyticity properties of the (dimensionless) spacelike physical quantities $\mathcal{D}(Q^2)$ such as (derivatives of) current correlators, structure functions, propagator dressing functions, etc. By the general principles of the quantum field theory (QFT) [1, 2], such physical quantities must be analytic (holomorphic) functions in the entire complex Q^2 -plane except on the negative semiaxis, i.e., $Q^2 \in \mathbb{C} \setminus (-\infty, -M_{\text{thr}}^2]$, where $M_{\text{thr}} \sim 10^{-1} \text{ GeV}$ is a threshold mass typical of the hadronic sector. Therefore, if these quantities $\mathcal{D}(Q^2)$ are to be evaluated as functions of $a(\kappa Q^2)$ (where $\kappa \sim 1$ is the renormalization scale parameter), the coupling $a(\kappa Q^2)$ should have qualitatively the same analyticity properties as $\mathcal{D}(Q^2)$.

The first model with holomorphic QCD coupling was constructed in [3–5]. In this model, named also Analytic Perturbation Theory (APT), the coupling $\mathcal{A}(Q^2)$ [the analytic analog of $a(Q^2)$] is obtained from $a(Q^2)$ by a minimal analytization approach. Namely, $\mathcal{A}(Q^2)$ is written as a dispersion integral involving the discontinuity function $\rho(\sigma) \equiv \text{Im}\mathcal{A}(-\sigma - i\epsilon)$, where $\rho(\sigma)$ is taken equal to its perturbative counterpart $\rho^{(\text{pt})}(\sigma) \equiv \text{Im}a(-\sigma - i\epsilon)$ on the entire negative axis in the Q^2 -plane ($\sigma > 0$), while the Landau singularities of $a(Q^2)$ on the positive Q^2 -axis were removed: $\rho(\sigma) = 0$ for $\sigma < 0$. Later on, the APT model was extended to analytic analogs $\mathcal{A}_\nu(Q^2)$ of the powers $a(Q^2)^\nu$ (for ν real, in general noninteger) in the works [6–9] (for reviews, see [10]). In these works, at the one-loop level of the underlying pQCD an explicit expression of $\mathcal{A}_\nu^{(1-\ell)}(Q^2)$ was obtained; at higher loop levels, the coupling \mathcal{A}_ν was expressed in terms of a series in ν -derivatives of $\mathcal{A}_\nu^{(1-\ell)}(Q^2)$. This extended model is called Fractional Analytic Perturbation Theory (FAPT).

Other models of analytic QCD were constructed later on. One such model is obtained by using the method of minimal analytization for the function $d \ln a(Q^2)/d \ln Q^2$, Refs. [11], which leads to a holomorphic coupling which diverges at $Q^2 = 0$. Another such model is, for example, the two-delta analytic QCD model [12] which uses for the discontinuity function $\rho(\sigma) \equiv \text{Im}\mathcal{A}(-\sigma - i\epsilon)$ the underlying pQCD values at high σ and parametrizes $\rho(\sigma)$ at low σ with two delta functions, and at high Q^2 the coupling $\mathcal{A}(Q^2)$ practically coincides with the pQCD coupling $a(Q^2)$. Analytization of general powers $a(Q^2)^\nu$ in general analytic QCD models of $\mathcal{A}(Q^2)$ was performed in Ref. [13].

In this work we are using FAPT to evaluate the dressing function $Z(Q^2)$ of the gluon propagator at low spacelike momenta in the Landau gauge, at the two-loop level of the underlying ($\overline{\text{MS}}$) pQCD. An analysis of this quantity was performed in Ref. [14] in the context of APT; however, at that time the analytization of noninteger powers was not yet known, and the analytization was performed using an alternative way at the one-loop level.

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In Sec. II we briefly review the FAPT model of analytization. In Sec. III A we present a two-loop pQCD calculation of the dressing function of the gluon propagator in the Landau gauge. In Sec. III B we include the nonperturbative effects, by introducing the gluon dynamical mass in the propagator, and by the FAPT analytization of the noninteger powers of the coupling in the dressing function. In Sec. IV we compare the results of the method with the results of the unquenched ($N_f = 3$) lattice data for the propagator and with pQCD results. In Sec. V we summarize our results.

II. FRACTIONAL ANALYTIC PERTURBATION THEORY (FAPT) MODEL

We present here a brief overview of the main ideas of the (Fractional) Analytic Perturbation Theory ((F)APT).

The pQCD coupling $a(Q^2) \equiv \alpha_s(Q^2)/\pi$, in the usual ($\overline{\text{MS}}$ -like) schemes, is running according to the perturbative renormalization group equation (pRGE) which has the beta function $\beta(a)$ in the form of a truncated power series up to n -loop order in a

$$\frac{\partial a(Q^2; \beta_2, \dots)}{\partial \ln Q^2} = - \sum_{j=0}^{n-1} \beta_j a^{j+2}(Q^2; \beta_2, \dots). \quad (1)$$

Here, the first two beta coefficients are universal

$$\beta_0 = \frac{1}{4} \left(11 - \frac{2}{3} N_f \right), \quad \beta_1 = \frac{1}{16} \left(102 - \frac{38}{3} N_f \right), \quad (2)$$

and the other coefficients β_k ($k \geq 2$) represent the (chosen) perturbative renormalization scheme. The integration of the pRGE (1) in the complex Q^2 -plane, with a given physical initial condition (at some high enough positive Q^2), gives in general a function $a(Q^2)$ which does not reflect the general analyticity properties of the spacelike QCD physical quantities in the complex Q^2 -plane: $a(Q^2)$ develops Landau singularities outside the timelike region $\mathbb{C} \setminus (-\infty, -M_{\text{thr}}^2]$, most often on the positive Q^2 axis: $0 < Q^2 < \Lambda_{\text{Lan}}^2$, where Λ_{Lan}^2 ($\sim 10^{-1} \text{ GeV}^2$) is the (Landau) branching point of these singularities. When the Cauchy theorem is applied to the function $a(Q'^2)/(Q'^2 - Q^2)$ in the Q'^2 complex plane on an appropriate closed contour which avoids all the cuts but encloses the pole $Q'^2 = Q^2$ (see Fig. 1), the following dispersion relation is obtained:

$$a(Q^2) = \frac{1}{\pi} \int_{\sigma=-\Lambda_{\text{Lan}}^2, -\eta}^{\infty} \frac{d\sigma \rho^{(\text{pt})}(\sigma)}{(\sigma + Q^2)}, \quad (\eta \rightarrow +0), \quad (3)$$

where $\rho^{(\text{pt})}(\sigma) = \text{Im} a(-\sigma - i\epsilon)$ is the discontinuity function of the pQCD coupling a along the entire cut axis. The

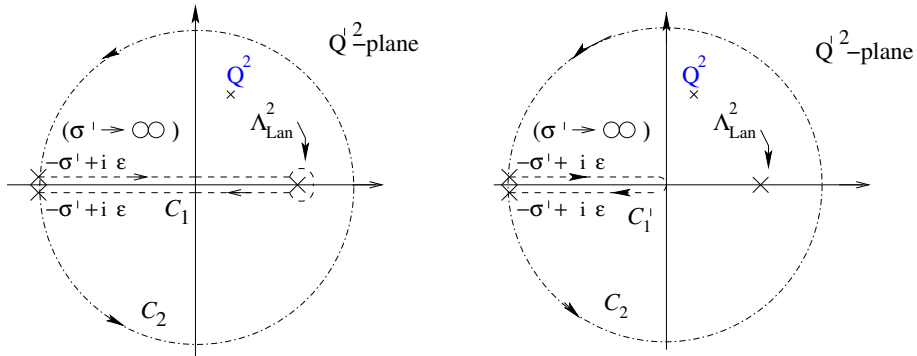


FIG. 1: Left-hand Figure: the integration path for the integrand $a_{\text{pt}}(Q'^2)/(Q'^2 - Q^2)$ leading to the dispersion relation (3) for $a_{\text{pt}}(Q^2)$. Right-hand Figure: the integration path for the same integrand, leading to the dispersion relation (4) for the APT coupling $\mathcal{A}^{(\text{APT})}(Q^2)$.

APT procedure [3] is the elimination of the contributions of the Landau cut $0 < (-\sigma) \leq \Lambda_{\text{Lan}}^2$, see Fig. 1. This gives the APT analytic analog $\mathcal{A}^{(\text{APT})}(Q^2)$ of $a(Q^2)$

$$\mathcal{A}^{(\text{APT})}(Q^2) = \frac{1}{\pi} \int_{\sigma=0}^{\infty} \frac{d\sigma \rho^{(\text{pt})}(\sigma)}{(\sigma + Q^2)}. \quad (4)$$

This procedure can be extended to the construction of the analogs $\mathcal{A}_n^{(\text{APT})}(Q^2)$ of n -integer powers $a(Q^2)^n$ [4, 5] and their combinations (see also [15]). The APT analogs of general powers a^ν (ν a real exponent) are obtained in the same way

$$\mathcal{A}_\nu^{(\text{FAPT})}(Q^2) = \frac{1}{\pi} \int_{\sigma=0}^{\infty} \frac{d\sigma \rho_\nu^{(\text{pt})}(\sigma)}{(\sigma + Q^2)} , \quad (5)$$

where

$$\rho_\nu^{(\text{pt})}(\sigma) = \text{Im } a^\nu(Q'^2 = -\sigma - i\epsilon) . \quad (6)$$

If the underlying pQCD running coupling $a(Q^2)$ is according to the one-loop pRGE, the corresponding explicit expressions for $\mathcal{A}_\nu^{(\text{FAPT})}$ exist and were obtained and used in Ref. [6]

$$\mathcal{A}_\nu(Q^2)^{(\text{FAPT}, 1-\ell.)} = \frac{1}{\beta_0^\nu} \left(\frac{1}{\ln^\nu(z)} - \frac{\text{Li}_{-\nu+1}(1/z)}{\Gamma(\nu)} \right) . \quad (7)$$

Here, $z \equiv Q^2/\Lambda^2$ and $\text{Li}_{-\nu+1}(z)$ is the polylogarithm function of order $-\nu+1$. Extensions of this FAPT approach to higher loops were performed by expanding the one-loop result in a series of derivatives with respect to the index ν [6–8]. We refer for reviews of FAPT to Refs. [10]. Mathematical packages for numerical calculations in APT and FAPT are given in Refs. [16].

We will use the numerical approach (5) for the calculation of the FAPT coupling, with the underlying pQCD coupling being the two-loop coupling [14, 17, 18] (see also [19, 20])

$$a(Q^2) = -\frac{1}{c_1} \frac{1}{[1 + W_{\mp 1}(z)]} , \quad (8)$$

where: $c_1 = \beta_1/\beta_0$; $Q^2 = |Q^2| \exp(i\phi)$; W_{-1} and W_{+1} are the branches of the Lambert function for $0 \leq \phi < +\pi$ and $-\pi < \phi < 0$, respectively; z is defined as

$$z = -\frac{1}{c_1 e} \left(\frac{|Q^2|}{\Lambda_{\text{L}}^2} \right)^{-\beta_0/c_1} \exp(-i\beta_0\phi/c_1) . \quad (9)$$

Here, Λ_{L} is the Lambert QCD scale. Since we will be interested in the behavior of the propagator at low momenta, we will use for simplicity $N_f = 3$ throughout, and $\Lambda_{\text{L}}(N_f = 3) = 0.581$ GeV. This value corresponds to $\Lambda_{\text{L}}(N_f = 5) = 0.322$ GeV (it corresponds to the $\overline{\text{MS}}$ scale $\bar{\Lambda}_{N_f=5} \approx 0.260$ GeV, used in Refs. [7, 8, 10]).

III. GLUON PROPAGATOR IN THE LANDAU GAUGE

A. Propagator in pQCD

Gluon propagator \mathcal{D} in the Landau gauge has the form

$$\mathcal{D}_{\mu\nu}^{ab}(Q) = \delta^{ab} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(-q^2) \quad (10)$$

where

$$D(Q^2) = \frac{Z(Q^2)}{Q^2} , \quad (11)$$

and in Eq. (11) we denoted $Q^2 \equiv -q^2 \equiv -(q_0)^2 + \vec{q}^2$ for the spacelike momenta ($Q^2 \in \mathbb{C} \setminus (-\infty, 0]$). Here, $Z(Q^2) \equiv Z(Q^2/\mu^2, a(\mu^2))$ is the dressing (residuum) function of the propagator. By the general principles of QFT, Z is an analytic (holomorphic) function of Q^2 in the Q^2 -complex plane $Q^2 \in \mathbb{C} \setminus (-\infty, 0]$, cf. Refs. [1, 2, 21]. We will assume (see the discussion in Sec. IV) that we are in the low-momentum region ($|Q| \equiv \sqrt{|Q^2|} \lesssim 1$ GeV) where there are three massless active quarks ($N_f = 3$). In this Subsection we present the two-loop pQCD expression of the dressing function $Z(Q^2)$. If the normalization is performed at a scale $\mu^2 = Q_0^2$, we have by the renormalization group invariance (Callan-Symanzik equation)

$$Z(Q^2) = \exp \left\{ \int_{a(Q_0^2)}^{a(Q^2)} \frac{\gamma(x)}{\beta(x)} dx \right\} , \quad (12)$$

where the anomalous dimension γ of Z , and beta function β , are taken at the two-loop level

$$\gamma(a) = -(\gamma_0 a + \gamma_1 a^2) \ , \quad \beta(a) = -(\beta_0 a^2 + \beta_1 a^3) \ , \quad (13)$$

with the coefficients β_0, β_1 given in Eq. (2), and

$$\gamma_0 = \frac{1}{8} \left(13 - \frac{4}{3} N_f \right) \ , \quad \gamma_1 = \frac{1}{16} \left(\frac{531}{8} - \frac{61}{6} N_f \right) \ . \quad (14)$$

We can rewrite

$$\frac{\gamma(a)}{\beta(a)} = \frac{\gamma_0}{\beta_0} \frac{1}{a} \frac{[1 + (\gamma_1/\gamma_0)a]}{[1 + (\beta_1/\beta_0)a]} = \frac{\gamma_0}{\beta_0} \left[\frac{1}{a} + \left(\frac{\gamma_1}{\gamma_0} - \frac{\beta_1}{\beta_0} \right) \frac{1}{(1 + (\beta_1/\beta_0)a)} \right] \ . \quad (15)$$

This then gives [we denote: $a \equiv a(Q^2)$, $a_0 \equiv a(Q_0^2)$]

$$\begin{aligned} Z(Q^2) &= \exp \left\{ \int_{a_0}^a \frac{\gamma(x)}{\beta(x)} dx \right\} = \exp \left\{ \frac{\gamma_0}{\beta_0} \int_{a_0}^a \left[\frac{1}{x} + \left(\frac{\gamma_1}{\gamma_0} - \frac{\beta_1}{\beta_0} \right) \frac{1}{(1 + (\beta_1/\beta_0)x)} \right] dx \right\} \\ &= \exp \left\{ \frac{\gamma_0}{\beta_0} \ln \left(\frac{a}{a_0} \right) + \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_0}{\beta_0} \right) \ln \left(\frac{1 + (\beta_1/\beta_0)a}{1 + (\beta_1/\beta_0)a_0} \right) \right\} \\ &= \left(\frac{a}{a_0} \right)^{\gamma_0/\beta_0} \left(\frac{1 + (\beta_1/\beta_0)a}{1 + (\beta_1/\beta_0)a_0} \right)^{\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_0}{\beta_0} \right)} \end{aligned} \quad (16)$$

This is the (formally exact) two-loop solution for the gluon dressing function. We can expand it in powers of a ; truncating at the NLO term then gives

$$Z(Q^2) = c_v [a^\nu(Q^2) + d_1 a^{\nu+1}(Q^2)] \ , \quad (17)$$

where c_v is a constant (independent of Q^2) to be fixed by a renormalization condition, and we have, according to Eq. (16)

$$\nu = \frac{\gamma_0}{\beta_0} \ , \quad d_1 = \left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1 \gamma_0}{\beta_0^2} \right) \ . \quad (18)$$

Numerically, $\nu = 0.5$ and $d_1 \approx 0.1076$ (when $N_f = 3$). The two-loop pQCD gluon propagator (10)-(11) is then

$$D_{\text{pt}}(Q^2) = \frac{c_v}{Q^2} [a^\nu(Q^2) + d_1 a^{\nu+1}(Q^2)] \ . \quad (19)$$

The truncated expansion (17) in powers of $a(Q^2)$ was performed for two reasons. Firstly, the analytization in the general analytic QCD frameworks [13] can be performed for powers of $a(Q^2)$, but not for the expression of the type (16).¹ Secondly, we will be working at the two-loop level; in the general n -loop case the coefficient d_1 of the expansion (17) depends only on the two-loop coefficients β_1 and γ_1 , while the full next coefficient d_2 involves also the three-loop (MS) coefficients β_2 and γ_2 which are not included in this work.

B. Nonperturbative effects in the gluon propagator

It is natural to consider the gluon with dynamical mass. Various studies of the gluon propagator in the Landau gauge using Dyson-Schwinger equations (DSEs) [22–26] indicate that in the infrared regime (low $|Q^2|$) the gluon acquires, via nonperturbative effects, a dynamical effective mass $M \approx 0.5$ -1 GeV. In these analyses,² gluon propagator in the Landau gauge tends to a nonzero constant in the infrared, $D(Q^2) \rightarrow \text{const.}$ when $Q^2 \rightarrow 0$ (decoupling scenarios), in

¹ Nonetheless, in (F)APT, the analytization can be performed on the entire expressions of the type (16), i.e., $a(Q^2)^\nu (1 + c_1 a(Q^2))^\mu$, cf. discussion in Ref. [15].

² In Refs. [27], Gribov-Zwanziger approach extended by condensates is applied, and qualitatively similar results for the gluon propagator are obtained as in the mentioned DSE analyses.

accordance with the presently available lattice data [28, 29].³ The simple version of the massive gluon propagator in the Landau gauge then has the form

$$D(Q^2) = \frac{Z(Q^2)}{Q^2 + M^2} , \quad (20)$$

where we assume that the dynamical mass M is Q^2 -independent. $Z(Q^2)$ is the dressing function discussed in the context of pQCD in the previous Subsection.

Nonperturbative physics must affect also the powers of the perturbative coupling $a(Q^2)$, as argued earlier. In this work we account for such effects by the analytization of the (two-loop) result (17)-(18) to FAPT, by replacing $a(Q^2)^{\nu+n} \mapsto \mathcal{A}_{\nu+n}^{(\text{FAPT})}(Q^2)$

$$Z(Q^2) = c_v \left[\mathcal{A}_{\nu}^{(\text{FAPT})}(Q^2) + d_1 \mathcal{A}_{\nu+1}^{(\text{FAPT})}(Q^2) \right] , \quad (21)$$

where $\mathcal{A}_{\nu}^{(\text{FAPT})}(Q^2)$ is given by Eqs. (5)-(6).

In conjunction with Eq. (20), the gluon propagator function $D(Q^2)$ of Eqs. (10)-(11) is now written as

$$D(Q^2; c_v, M^2) = \frac{c_v}{Q^2 + M^2} \left[\mathcal{A}_{\nu}^{(\text{FAPT})}(Q^2) + d_1 \mathcal{A}_{\nu+1}^{(\text{FAPT})}(Q^2) \right] . \quad (22)$$

The introduction of the dynamical effective gluon mass in Eq. (20) represents an inclusion of the nonperturbative effects which depend on the quantity considered,⁴ which in this case is gluon propagator. This is analogous to the approaches in Refs. [32–37] (also: [38, 39]) where nonperturbative contributions are introduced in the specific considered observables.

IV. NUMERICAL RESULTS

Before presenting our results, we note that our results should be compared with the lattice data for the unquenched case ($N_f = 3$), Ref. [29], in the available interval of $Q \equiv \sqrt{Q^2}$: $0.1 \text{ GeV} < Q \lesssim 10 \text{ GeV}$. This is so because the (F)APT formalism requires $N_f \geq 3$. Namely, the thresholds in the (F)APT formalism are understood to be implemented in general at $Q^2 = m_q^2$ (m_q is the corresponding quark mass) in the underlying pQCD coupling $a(Q^2)$, and the latter coupling has Landau singularities at energies $0 < Q^2 \lesssim m_s^2$ (m_s is the strange quark mass) and even at $Q^2 > m_s^2$.

We work with the gluon propagator (22), which has FAPT-holomorphic dressing function (21) and a dynamical gluon mass M , Eq. (20). The free parameters are c_v and M . We are choosing certain two points of the (low- Q) lattice data and adjust the free parameters (c_v and M) so that our curve goes through these two points; the two chosen points are also varied, so as to obtain (visually) the best curve. This approach is applied in the analytic (FAPT) and in the pQCD case.

In this context, we mention that a variant of APT was applied to the dressing function in Ref. [14], but in a more naive manner since the FAPT approach was not known at the time; and the dynamical gluon mass effect was not included.

In Fig. 2, we compare with the lattice data at low positive Q^2 ($Q \equiv \sqrt{Q^2}$) when fixing $M = 0$ in $D(Q^2)$ of Eq. (22), while the other parameter c_v is fixed by a lattice point. We include also a (two-loop) pQCD result (19), where in $a(Q^2)$ we use the same Lambert scale: $\Lambda_L(N_f = 3) = 0.581 \text{ GeV}$.

In Fig. 3 both parameters, M and c_v , are varied in Eq. (22) so as to get the best possible agreement with the lattice data. For additional comparisons, a representative pQCD curve with $M \neq 0$ [i.e., Eq. (20) with (17)] is included, where we use the aforementioned Lambert scale.

In Fig. 2 we see that the analytization alone (and $M = 0$) gives us results which are good down to approximately 0.8 GeV; this is an improvement with respect to pQCD. But we wanted to go beyond and see what is the effect of including a (small) dynamical effective mass M of gluon, Eqs. (20) and (22). We find that the best mass parameter is $M^2 \approx 0.382 \text{ GeV}^2$, see Fig. 3. It turns out that this value is consistent with the values of M obtained in Refs. [40].⁵

³ Other DSE studies [30] indicate that $D(Q^2) \rightarrow 0$ (when $Q^2 \rightarrow 0$) is also a solution to DSEs (scaling scenarios), and in Ref. [31] both scenarios (scaling and decoupling) are discussed.

⁴ Unlike the analytization $a^\nu \mapsto \mathcal{A}_\nu$ which is independent of the quantity considered.

⁵ In Refs. [40], $M(Q^2)$ and $Z(Q^2)$ were considered as Q^2 -dependent functions with specific Ansätze, and the resulting propagator was fitted to the lattice results.

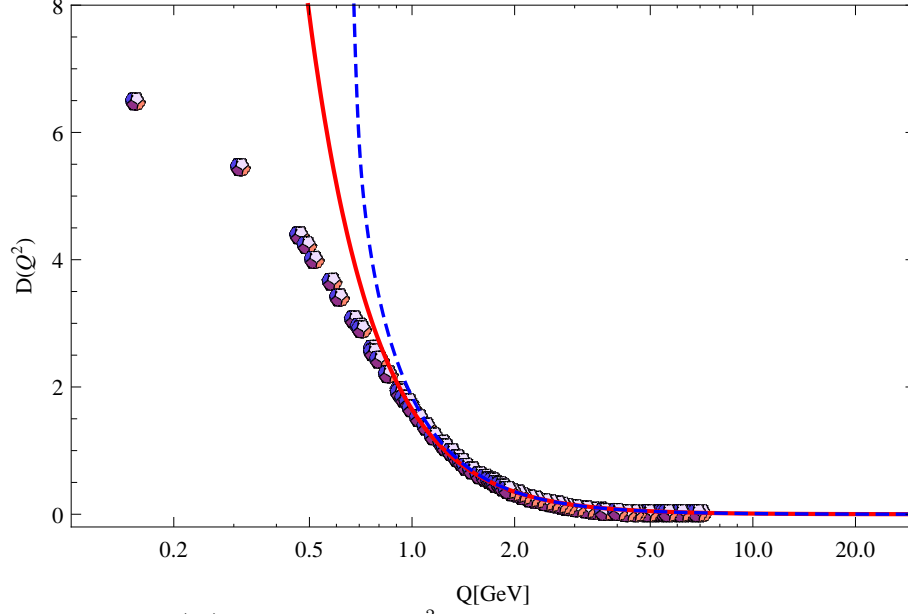


FIG. 2: Analytic gluon propagator (22), in units of GeV^{-2} , with the dynamical effective gluon mass parameter $M = 0$ and $c_v = 5.10$ (continuous line), in comparison with unquenched lattice data taken from Ref. [29], where $N_f = 2 + 1$. Further, the (two-loop) pQCD result (19) is presented as well, with $c_v = 4.55$ (dashed line).

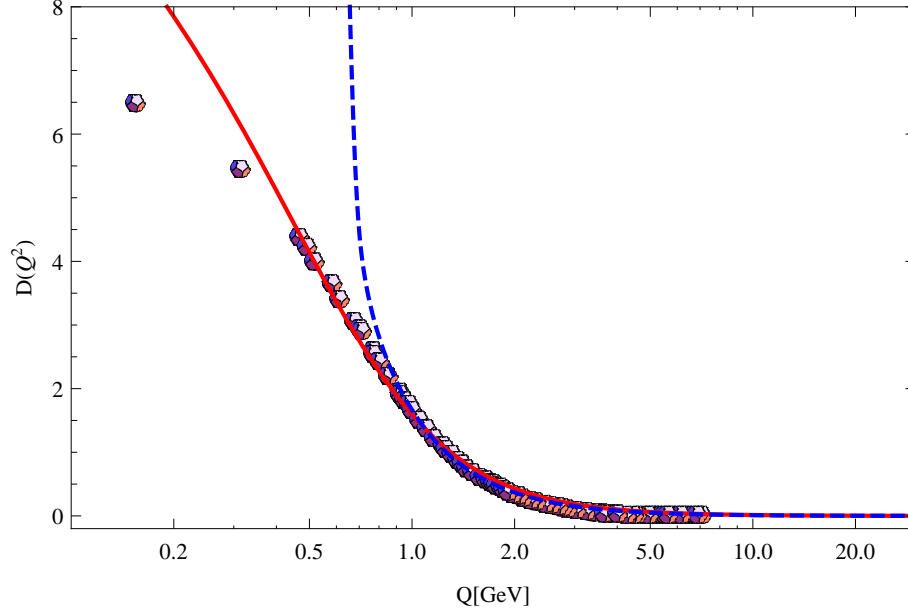


FIG. 3: The same as in Fig. 2, but now the dynamical effective gluon mass parameter M is nonzero: the analytic propagator (22) (continuous line) has $M^2 = 0.382 \text{ GeV}^2$ and $c_v = 6.81$. The pQCD result (dashed line), i.e., Eq. (20) with (17), has $M^2 = 0.211 \text{ GeV}^2$ and $c_v = 4.98$.

We see from Fig. 3 that the massive FAPT-analytic version is applicable for momenta down to $Q \approx 0.4 \text{ GeV}$ ($Q^2 \approx 0.15 \text{ GeV}^2$), but not below that. This is consistent with the conclusions about the applicability of the APT approach in Bjorken Polarized Sum Rule at low Q , Ref. [41], where the authors included other nonperturbative effects via higher-twist terms.

We note that in our approach, when we want to reproduce lattice results for lower momenta Q , the mass parameter M is getting bigger and is accompanied with a worse behavior at higher Q values. Therefore, we intend to improve this approach in the future, by using a Q^2 -dependent dynamical mass $M(Q^2)$ of the gluon.

If we performed our analysis with the one-loop FAPT, this would affect in the present approach only the dressing

function $Z(Q^2)$ and not the mass M^2 , i.e., in Eq. (21) we would have only one term. This would give us almost the same result as in the two-loop case of FAPT, and the resulting curves would be almost indistinguishable from the solid curves in Figs. 2,3. This is due to the small value of $d_1 \approx 0.1076$ and due to the very good hierarchy of FAPT couplings: $|\mathcal{A}_{\nu+1}^{(\text{FAPT})}(Q^2)| \ll |\mathcal{A}_{\nu}^{(\text{FAPT})}(Q^2)|$, true even at low $|Q^2|$, cf. Refs. [10]. In the context of DSEs, the effect of two-loop effects was investigated in Ref. [42].

V. SUMMARY

In this work, we evaluated the gluon propagator in the Landau gauge at low spacelike momenta Q^2 . We used the two-loop solution of the Callan-Symanzik equation for the dressing function $Z(Q^2)$. The nonperturbative effects were incorporated in the form of the analytization procedure $a^\nu(Q^2) \mapsto \mathcal{A}_\nu(Q^2)$ for the (noninteger) powers of the QCD coupling $a(Q^2)$, within Fractional Analytic Perturbation Theory (FAPT) with $N_f = 3$; and by incorporation of a constant dynamical effective gluon mass M in the propagator. The obtained expression Eq. (22) has two free parameters: the normalization constant c_v and the dynamical gluon mass M . We compared the obtained results with the unquenched lattice results ($N_f = 3$) at low positive Q^2 . In comparison with pQCD results, the analytization clearly improved the behavior of the propagator at low Q . The additional introduction of the dynamical effective gluon mass further improved the low- Q behavior. We used the FAPT with $N_f = 3$, because in FAPT it is apparently not possible to define unambiguously the theory for $N_f < 3$.

The main results can be summarized as follows:

1. We performed an analytization procedure (numerical FAPT) for the dressing function of gluon propagator in the Landau gauge. The dressing function was obtained from the two-loop Callan-Symanzik equation, and this (pQCD) procedure introduces one free parameter c_v which is an overall normalization constant.
2. In addition, we introduced a (constant) dynamical effective gluon mass M in the propagator, as suggested by various DSE studies of gluon propagator in the Landau gauge.
3. We compared the obtained results with the unquenched lattice data of the propagator, by varying the free parameters c_v and M .
4. In the nonmassive ($M = 0$) case, the analytic gluon propagator is in agreement with the unquenched lattice data down to $Q^2 \approx 0.6 \text{ GeV}^2$ ($Q \equiv \sqrt{Q^2} \approx 0.8 \text{ GeV}$), while the massive ($M > 0$) analytic gluon propagator agrees with the lattice data down to $Q^2 \approx 0.15 \text{ GeV}^2$ ($Q \approx 0.4 \text{ GeV}$).
5. The values that we found for our fit are $c_v = 5.10$ for the nonmassive case; and $c_v = 6.81$ and $M^2 = 0.382 \text{ GeV}^2$ ($M \approx 0.62 \text{ GeV}$) for the massive case, where this value of the dynamical effective gluon mass M is similar to the values found in the literature.

We intend to continue this work in various directions: perform the analytization procedure within other QCD models such as the two-delta analytic QCD model of Ref. [12] and the analytic QCD models with effective mass in the coupling (cf. Refs. [43–48]); allow Q^2 -dependence in the dynamical effective gluon mass M ; and compare with DSE results (numerically and theoretically).

Acknowledgments

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